MEASUREMENT OF THE TEMPERATURE FIELD DURING THE PLANAR STEADY-STATE FLOW OF A METAL

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The experimental determination of the temperature of a metal during its impulsive high-rate deformation is made very difficult due to the fact that local sensors cannot be used to measure temperature in this case, since they violate the integrity of the specimen. Even if they have little effect on such flow parameters as pressure, density, or the velocity field, cavities, slits, dielectrics, etc., inside the metal undergoing impulsive deformation usually distort the temperature field in an uncontrollable manner. Thus, if the problem is to measure temperature in the interior of a specimen, then the specimen itself should act as the sensor. When the thermocouple method of measurement is used, such specimen-sensors can be obtained if one joins two metals with similar mechanical characteristics but different thermoelectric characteristics. Here, the goal is to avoid distortions of the temperature field by the interface between the metals.

In the present study, we describe a thermoelectric method of measuring the temperature field during the planar steady-state flow of a metal. An important aspect of the method is that it does not introduce distortions into the measured temperature field. In a special case, the method can be used to measure the temperature of a weld during explosive welding. Zakhavenko [1] and Mikhailov et al. [2] employed the Seebeck effect to measure temperature in the explosive-welding regime. However, only the residual temperature of the weld was recorded in [1] and [2]. In [2], a complex probe was introduced into the measurement zone. This seriously complicated the interpretation of the resulting data.

We will examine the collision of two infinite plane jets. One of the jets consists of metal 1 (Fig. 1), while the other (bimetallic) consists of metals 1 and 2. We will designate the free boundaries of the flow as G_i ($i = 1, 2, 3, 4$); the boundary dividing the metals will be designated as Γ; the coordinates, reckoned along these boundaries from left to right, are g_i and γ, respectively. Let a sensor (which we will henceforth conditionally refer to as a voltmeter) be attached to points A and B with the coordinates g_1 and g_2. This sensor measures the voltage between the given points $V(g_1, g_2)$. We will show that if the distribution $V(g_1, g_2)$ is measured for a fixed g_1, then with a known field of flow velocity $\mathbf{u}$ the temperature distribution along Γ $T(\gamma)$ is determined unambiguously.

In actual experiment, such a pattern can be obtained as follows (Fig. 2). We use a plane explosive charge to project a metallic plate 1 against a stationary bimetallic plate. The voltmeter was connected by fixed leads to the middle region of the second plate and the edge of the first plate. Assuming the plates are of sufficient dimensions, in the coordinate system connected with the stagnation point O the flow of metal away from the edges can be considered planar and steady after the elapse of a certain amount of time from the beginning of the collision. In this system, the measurement lead rigidly attached to the bottom plate and moving together with the metal passes along G_2 from $g_2 = -\infty$ to $g_2 = +\infty$. The experimentally measured dependence of the voltage on the voltmeter on time $V(t)$ gives the distribution $V(+\infty, g_2)$.

The deformation of the metal establishes a certain distribution of temperature T, while the thermoelectric effect produces the current density $\mathbf{j}$ and the magnetic $\mathbf{H}$ and electric $\mathbf{E}$ fields. We introduce a Cartesian coordinate system with the z axis perpendicular to the plane of the flow. We will assume that in the system connected with the stagnation point, the distributions $T(x, y)$, $\mathbf{j}(x, y)$, $\mathbf{H}(x, y)$, and $\mathbf{E}(x, y)$ satisfy conditions of stationariness and two-dimensionality; here, $\mathbf{E}$ and $\mathbf{j}$ will have only x- and y-components, while $\mathbf{H}$ will have only a z-component: $\mathbf{H} = \mathbf{e}_z H$.

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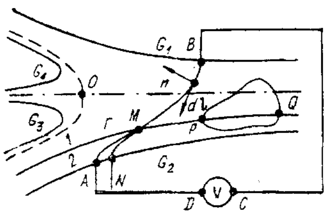

Fig. 1

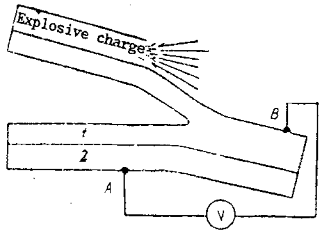

Fig. 2

Let the section of the measurement lead AD be made of metal 2, and let section BC be made of metal 1. The voltmeter is connected to points D and C by the lead from metal 2 (see Fig. 1). We will assume that the temperature at points C and D is equal to the ambient temperature T_0. We isolate a closed loop L lying in a certain plane z = const and passing along the measurement leads from A to B through the voltmeter. Meanwhile, while passing from B to A through the metal, the leads intersect Γ at a certain point M (counterclockwise is the positive direction). The relationship between the measured voltage and the distribution T(γ) can be obtained as follows. On the one hand,

$$\oint_L \mathbf{E}\,d\mathbf{l} = \int_A^D \mathbf{E}\,d\mathbf{l} + V(g_1, g_2) + \int_C^B \mathbf{E}\,d\mathbf{l} + \int_B^M \mathbf{E}\,d\mathbf{l} + \int_M^A \mathbf{E}\,d\mathbf{l}$$

($d\mathbf{l}$ is an element of length along L). On the other hand, by virtue of the stationariness of the problem, $\oint_L \mathbf{E}\,d\mathbf{l} = 0$. This means that

$$V(g_1, g_2) = -\int_A^D \mathbf{E}\,d\mathbf{l} - \int_C^B \mathbf{E}\,d\mathbf{l} - \int_B^M \mathbf{E}\,d\mathbf{l} - \int_M^A \mathbf{E}\,d\mathbf{l}. \quad (1)$$

With allowance for the thermoelectric effect, the expression for Ohm's law for a particle of metal i moving with the velocity $\mathbf{u}$ has the form

$$\mathbf{E} = -\frac{1}{e}\nabla\mu + \frac{1}{\sigma_i}\mathbf{j} + s_i\nabla T - \mu_0[\mathbf{uH}], i = 1, 2. \quad (2)$$

Here, e is the electron charge; μ is the chemical potential; σ_i and s_i are the conductivity and absolute thermoelectric coefficient of the metal i; μ_0 is the magnetic constant. We will henceforth assume that σ_i are constant, while s_i depend only on temperature. Since the resistance of the voltmeter is much greater than the resistance of the region occupied by the metal, the current flowing through the measurement leads is negligibly small and there is no magnetic field outside the metal. Then, inserting (2) into (1), we obtain

$$\begin{aligned} V = &\int_A^D \frac{1}{e}\nabla\mu\,d\mathbf{l} + \int_C^B \frac{1}{e}\nabla\mu\,d\mathbf{l} + \int_B^M \frac{1}{e}\nabla\mu\,d\mathbf{l} + \int_M^A \frac{1}{e}\nabla\mu\,d\mathbf{l} - \\ &- \int_{T_0}^{T_M} (s_1 - s_2)\,dT - \int_B^M \frac{1}{\sigma_1}\mathbf{j}\,d\mathbf{l} - \int_M^A \frac{1}{\sigma_2}\mathbf{j}\,d\mathbf{l} + \int_B^A \mu_0[\mathbf{uH}]\,d\mathbf{l} \end{aligned} \quad (3)$$

(T_M is the temperature at the point M).

Since the integral of ∇μ over the closed contour is equal to zero, and since ∇μ equals zero on the section from D to C (this section consists solely of metal at the constant temperature T_0), then the sum of the first four terms in (3) is equal to zero. In the plane case, it follows from the equation $\mathbf{j}$ = rot $\mathbf{H}$ that

$$\mathbf{j}\,d\mathbf{l} = -(\partial H/\partial n)dl \quad (4)$$

($\mathbf{n}$ is an outer unit normal to $d\mathbf{l}$).

Since u has only x- and y-components and since $\mathbf{H}$ has only z components, then

$$[\mathbf{uH}]d\mathbf{l} = -H\mathbf{un}\, dl. \tag{5}$$

Considering (4-5), we change (3) to the form

$$V = -\int_{T_0}^{T_M} (s_1 - s_2)\, dT + \mu_0 \int_B^M \left(\frac{1}{\mu_0\sigma_1} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl + \mu_0 \int_M^A \left(\frac{1}{\mu_0\sigma_2} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl. \tag{6}$$

The physical significance of this expression can be explained as follows: the first term describes the direct effect of the temperature gradient along the path of integration, while the second and third terms describe the change in the magnetic flux across L. This change is a reflection of two processes: the gradient terms give the diffusion flux of the field over the curve BA, while the terms with $H\mathbf{u}$ describe the transport of the magnetic field through BA by the moving metal.

Equation (6) makes it possible to use the measured voltage to determine the temperature T_M at any point of the boundary Γ if we know H(x, y). The distribution of the magnetic field in the metal can be found from the solution of the following two problems. Having subjected Eq. (2) to the operation rot and taking into account that rot $\mathbf{E} = 0$, rot $\nabla\mu = 0$, $\mathrm{rot}(s_i\nabla T) = 0$, rot $\mathbf{j} = -\Delta\mathbf{H}$, rot $[\mathbf{uH}] = -\mathrm{div}\,(H\mathbf{u})$, we find that in each metal the magnetic field satisfies the equation

$$(1/\mu_0\sigma_i)\Delta H - \mathrm{div}(H\mathbf{u}) = 0,\ i = 1,\ 2. \tag{7}$$

Equation (7) needs to be solved with two boundary conditions. For the region occupied by metal 2, H = 0 on G_2. This follows from the finiteness of the component j which is tangential to the external boundary and from the absence of a magnetic field outside the metal. We find the second boundary condition after having calculated $\partial V/\partial g_2$. For this, we take N on G_2, N having the coordinate g_2 + sine dg_2. We isolate the contour L_1, coinciding on the sections DC, CB, and BM with L and passing from M to N and from N to D along certain curves lying within L. Following (6), we write

$$V(g_1, g_2 + dg_2) = -\int_{T_0}^{T_M} (s_1 - s_2)\, dT + \mu_0 \int_B^M \left(\frac{1}{\mu_0\sigma_1} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl + \\ + \mu_0 \int_M^N \left(\frac{1}{\mu_0\sigma_2} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl. \tag{8}$$

It follows from (7) that

$$\oint_{ANMA} \left(\frac{1}{\mu_0\sigma_2} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl = 0.$$

Then from (6) and (8) we obtain

$$V(g_1, g_2 + dg_2) - V(g_1, g_2) = \mu_0 \int_A^N \left(\frac{1}{\mu_0\sigma_2} \nabla H - H\mathbf{u}\right)\mathbf{n}\, dl.$$

Since H = 0 on G_2, we can use the latter to obtain the second boundary condition needed to find H(x, y) in the region occupied by metal 2;

$$\left.\frac{\partial H}{\partial n}\right|_{G_2} = \sigma_2 \frac{\partial V}{\partial g_2}. \tag{9}$$

Generally speaking, we are dealing with an ill-conditioned problem when we attempt to solve Eq. (7) for the region occupied by metal 2 with boundary conditions on G_2 H = 0 and (9). However, assuming that the sought solution is smooth and using regularization methods, it is possible to obtain a solution which is continuously dependent on the error of the boundary conditions [3]. Having then solved the problem, we find H(x, y) in the region occupied by metal 2 and, in particular we find the distribution of the magnetic field along Γ H(γ). By virtue of the continuity of the magnetic field on Γ, the resulting H(γ) serves as the boundary condition to determine H(x, y) in the region occupied by metal 1. For this, we need to solve (7) with i = 1 and the second boundary condition — which is analogous to the first condition used in the calculation of H in the region occupied by metal 2: H = 0 on G_1, G_3, G_4. This problem is well conditioned. After we calculate H(x, y) throughout the region of metal flow, we can use (6) to find T_M. Moving point M along Γ and solving (6) for each position M, we obtain the temperature distribution along the interface of the metals T(γ). If the mechanical characteristics of the metals in the above-described flow are similar and if the boundary Γ does not introduce significant distortions in the velocity and temperature fields, then by determining T(γ) for different positions of the interface in the same flow configuration we can construct T(x, y) for the entire region. In a particular case, we can proceed in this manner to measure the temperature of a weld in the explosive-welding regime. To do this, the bottom plate should consist entirely of metal 2, rather than being bimetallic.

We make the following observation. In the numerical solution of boundary-value problems, the region in which H(x, y) is calculated is cut off a finite distance from the stagnation point upstream and downstream. If the boundaries obtained from this procedure are located sufficiently far from the stagnation point, it can be assumed that H = 0. In fact, Eqs. (7) show that the magnetic field diffuses through the metal (first term) and is simultaneously entrained by the moving medium (second term). It follows from (7) that the below expression is valid for any closed contour lying entirely within the metal and not containing the boundary Γ

$$\oint\left(\frac{1}{\mu_0\sigma_i}\nabla H - H\mathbf{u}\right)\mathbf{n}\,dl = 0,\ i = 1,\ 2. \tag{10}$$

This means that there are no magnetic-field sources for Eqs. (7) within the uniform region of the metal. If we isolate a contour containing the section PQ of the boundary Γ (see Fig. 1), then it is not hard to show that

$$\oint\left(\frac{1}{\mu_0\sigma}\nabla H - H\mathbf{u}\right)\mathbf{n}\,dl = -\int_P^Q\left(\frac{1}{\mu_0\sigma_1}\frac{\partial H}{\partial n_1} + \frac{1}{\mu_0\sigma_2}\frac{\partial H}{\partial n_2}\right)d\gamma \tag{11}$$

(n_1 and n_2 are unit vectors perpendicular to Γ and directed inward from the regions occupied by metals 1 and 2, respectively). Taking into account the continuity of the tangential component of **E** with the crossing of the interface and using (2) and (4), we find that on Γ

$$\frac{1}{\sigma_1}\frac{\partial H}{\partial n_1} + \frac{1}{\sigma_2}\frac{\partial H}{\partial n_2} = (s_1 - s_2)\frac{\partial T}{\partial \gamma}. \tag{12}$$

Comparing (11) and (12), we see that the sources of the magnetic field for (7) are found on the interface where there is a temperature gradient along Γ. It can be determined that the magnetic flux created in 1 sec per unit length of Γ will be $J = (s_1 - s_2)\partial T/\partial\gamma$. The temperature gradient is concentrated mainly in the region around the stagnation point, where the metal undergoes intensive deformation. The magnetic field created in this region diffuses primarily toward the external boundaries of the flow, where H = 0, and is simultaneously entrained by the moving metal. As follows from (7), the characteristic times over which the field reaches the free boundaries $\tau_1 \sim \mu_0\sigma_1\delta_1^2$ and $\tau_2 \sim \mu_0\sigma_2\delta_2^2$ (δ_1 and δ_2 are the thicknesses of metals 1 and 2). The thermal conductivity of the metals leads to a decrease in $\partial T/\partial\gamma$ with increasing distance from the stagnation point. At a sufficient distance from this point, it can be assumed that $\partial T/\partial\gamma = 0$ and that there are no magnetic-field sources. The field entrained by the moving metal diffuses to the external boundaries and disappears. The condition H = 0 will be satisfied for certain when

$$|g_1|, |g_2| \gg \max\left(\frac{u_0a^2}{\varkappa_1}, \frac{u_0a^2}{\varkappa_2}, u_0\tau_1, u_0\tau_2\right),$$

where u_0 is the velocity of the metal at infinity; a is the characteristic dimension of the region in which the metal undergoes deformation; $\varkappa_1$ and $\varkappa_2$ are the diffusivities of the metals. If there is a large difference in the conductivities of the materials, then this condition will be too rigid – since the velocity of the magnetic field as it leaves the metal is determined by the material with the lesser conductivity.

The present method, requiring measurement of temperature with an arbitrary velocity field $\mathbf{u}(x, y)$ entails a fairly large amount of numerical calculation. This is because it is necessary to solve the boundary-value problem twice in order to find $H(x, y)$. The method can be simplified appreciably if the velocity field can be described in the model of an ideal incompressible fluid. As was shown in [4, 5], by comparing the empirically determined pressure field in copper during explosive welding and the results calculated from the model of an ideal incompressible fluid, the latter can be used as a first approximation to describe metal flow under such conditions. We will examine the case in which the densities of the metal are identical. Then the flow will be continuous throughout the entire region. We introduce the velocity potential φ and the stream function ψ:

$$\partial\varphi/\partial x = u_x,\ \partial\varphi/\partial y = u_y,\ \partial\psi/\partial x = -u_y,\ \partial\psi/\partial y = u_x.$$

On the plane of the complex potential $\Phi = \varphi + i\psi$, the region occupied by metal 2 becomes a straight band $\psi(G_2) \leq \operatorname{Im}\Phi \leq \psi(\Gamma)$ [$\psi(G_2)$ and $\psi(\Gamma)$ are values of the stream function on G_2 and Γ, respectively]. In order to avoid differentiating the empirically measured quantity V when we obtain boundary condition (9), rather than seeking $H(x, y)$ we attempt to find the auxiliary function $F_2(\varphi, \psi)$, defined in the region occupied by metal 2

$$\frac{\partial F_2}{\partial\varphi} = H(\varphi, \psi),\ \frac{\partial F_2}{\partial\psi} = \int\limits_{-\infty}^{\varphi} \frac{\partial H}{\partial\psi}\, d\varphi. \tag{13}$$

Inserting (13) into (7) and integrating, we obtain

$$\partial^2 F_2/\partial\varphi^2 + \partial^2 F_2/\partial\psi^2 - \mu_0\sigma_2\partial F_2/\partial\varphi = f(\psi) \tag{14}$$

[$f(\psi)$ is an arbitrary function]. At $\varphi \to \infty$, the magnetic field disappears and, thus, $\partial F_2/\partial\varphi$ and $\partial^2 F_2/\partial\varphi^2$ approach zero. We will isolate a contour formed by two arbitrary streamlines $\psi = \psi_1$ and $\psi = \psi_2$ and closed by the sections $\varphi = \text{const}$ at $\varphi \to \pm\infty$. Since there is no convective transport of the magnetic field across the streamlines ($\mathbf{un} = 0$), then we find from (10) and (13) that

$$\left.\frac{\partial^2 F_2}{\partial\psi^2}\right|_{\varphi\to\infty} = \frac{\partial}{\partial\psi}\int\limits_{-\infty}^{\infty} \frac{\partial H}{\partial\psi}\, d\varphi = 0.$$

This means that $f(\psi) \equiv 0$. Since F_2 is determined to within a constant, it can be assumed that $F_2(\varphi, \psi(G_2)) = 0$. It follows from (9) and (12) that on G_2

$$\partial F_2/\partial\psi = -\sigma_2 V(g_1(\varphi, \psi),\ g_2(\varphi, \psi)). \tag{15}$$

The problem of finding $F_2(\varphi, \psi)$ is solved by the source method. The solution of Eq. (14) for a linear source of power q located at the origin of the coordinates in the plane (φ, ψ) has the form [6]

$$\varkappa = \frac{q}{2\pi}\exp\left(\frac{\mu_0\sigma_2\varphi}{2}\right) K_0\left(\frac{\mu_0\sigma_2\sqrt{\varphi^2+\psi^2}}{2}\right)$$

(K_0 is the zeroth-order Bessel function of the imaginary argument, this function being of the second kind).

We will place sources with the density $\rho(\varphi)$ on Γ and place sources with the density $\rho(\varphi)$ symmetrically relative to G_2. In this case,

$$F_2(\varphi, \psi) = \int\limits_{-\infty}^{\infty} \frac{\rho(\varphi_0)}{2\pi}\exp\left[\frac{\mu_0\sigma_2(\varphi-\varphi_0)}{2}\right] \times \tag{16}$$

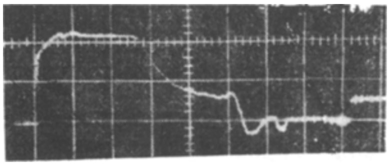

Fig. 3

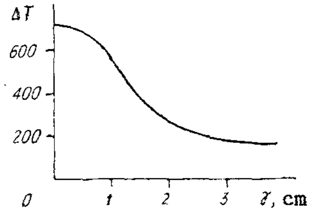

Fig. 4

$$\times K_0\left(\frac{\mu_0\sigma_2}{2}\sqrt{(\varphi-\varphi_0)^2+[\psi-\psi(\Gamma)]^2}\right)d\varphi_0-\int_{-\infty}^{\infty}\frac{\rho(\varphi_0)}{2\pi}\exp\left[\frac{\mu_0\sigma_2}{2}(\varphi-\varphi_0)\right]\times$$

$$\times K_0\left(\frac{\mu_0\sigma_2}{2}\sqrt{(\varphi-\varphi_0)^2+[\psi+\psi(\Gamma)-2\psi(G_2)]^2}\right)d\varphi_0.$$

Due to the symmetry of the location of the sources relative to G_2, the boundary condition $F_2(\varphi, \psi) = 0$ at $\psi = \psi(G_2)$ is satisfied automatically. Differentiating (16) with respect to ψ and taking (15) into account, to determine the function $\rho(\varphi)$ we obtain a Fredholm equation of the first kind:

$$\int_{-\infty}^{\infty}\frac{\rho(\varphi_0)}{2\pi}\frac{\mu_0\sigma_2[\psi(\Gamma)-\psi(G_2)]\exp\left[\frac{\mu_0\sigma_2}{2}(\varphi-\varphi_0)\right]}{\sqrt{(\varphi-\varphi_0)^2+[\psi(\Gamma)-\psi(G_2)]^2}}\times$$

$$\times K_1\left(\frac{\mu_0\sigma_2}{2}\sqrt{(\varphi-\varphi_0)^2+[\psi(\Gamma)-\psi(G_2)]^2}\right)d\varphi_0=-\sigma_2 V(g_1(\varphi,\psi(G_2)),\ g_2(\varphi,\psi(G_2))).$$

Using regularization methods, we can take this equation and find $\rho(\varphi)$. We can then use (16) to find $F_2(\varphi, \psi)$. We introduce the function $F_1(\varphi, \psi)$, determined by the relations

$$\frac{\partial F_1}{\partial\varphi}=H(\varphi,\psi),\ \frac{\partial F_1}{\partial\psi}=\int_{-\infty}^{\varphi}\frac{\partial H}{\partial\psi}d\varphi. \tag{17}$$

for the region occupied by metal 1. In accordance with (14), we obtain

$$\partial^2F_1/\partial\varphi^2+\partial^2F_1/\partial\psi^2-\mu_0\sigma_1\partial F_1/\partial\varphi=0. \tag{18}$$

Since $H = 0$ on G_1, G_3, and G_4, the function $F_1 = \text{const}$ on these boundaries and can be set equal to zero. Due to the continuity of the magnetic field on Γ, it follows from (13) and (17) that

$$F_1(\varphi,\psi(\Gamma))=F_2(\varphi,\psi(\Gamma))+\text{const}. \tag{19}$$

We find from (17) that $\partial F_1/\partial\psi \to 0$ at $\varphi \to -\infty$, while since $F_1 = 0$ on the external boundaries, then $F_1(\varphi, \psi(\Gamma)) \to 0$ at $\varphi \to -\infty$. Similarly, $F_2(\varphi, \psi(\Gamma)) \to 0$ at $\varphi \to -\infty$ so that const = 0 in (19). Thus, the value of $F_2(\varphi, \psi(\Gamma))$ serves as the closing boundary condition (19) to find $F_1(\varphi, \psi)$. The problem of solving Eq. (18) with the above-indicated boundary conditions is well conditioned.

After determining F_1 and F_2, in principle we could use (13) and (17) to obtain H, while (6) could be used to find the temperature. However, it is more convenient to take another approach. In fact, integrating (12) along Γ and considering (13), (17), we see that the temperature at an arbitrary point M can be found from the relation

$$\int_{T_0}^{T_M}(s_1-s_2)\,dT=\frac{1}{\sigma_2}\lim_{\psi\to\psi(\Gamma)}\frac{\partial F_2(\varphi(M),\psi)}{\partial\psi}-\frac{1}{\sigma_1}\lim_{\psi\to\psi(\Gamma)}\frac{\partial F_1(\varphi(M),\psi)}{\partial\psi}. \tag{20}$$

The method is made even simpler when $\sigma_1 \gg \sigma_2$. In this case, we can ignore the second term in the right side of (20), so that it is no longer necessary to find $F_1(\varphi, \psi)$. This situation is realized, for example, when metal 2 is constantan and metal 1 is copper ($\sigma_1/\sigma_2 \simeq 30$).

TABLE 1

v_C, m/sec	γ, deg				
	12	14	18	20	23
	T				
Copper–constantan					
930	— —	— —	720 180	— —	— —
1000	— —	— —	750 240	— —	— —
1340	840 250	— —	— —	— —	— —
1500	940 280	— —	— —	— —	— —
2200	— —	— —	— —	1200 550	— —
2700	— —	1300 470	— —	— —	— —
Manganin–constantan					
980	— —	— —	1030 400	— —	— —
1340	— —	— —	1020 400	— —	— —
1480	— —	— —	— —	— —	980 350

In another special case, when the temperature of the weld is measured in the explosive-welding regime with a symmetrical collision of metals having the same conductivity (for example, manganin and constantan), it also suffices to determine just $F_2(\varphi, \psi)$. Then, due to the symmetry of the problem, $F_1(\varphi, \psi) = F_2(\varphi, 2\psi(\Gamma) - \psi)$ and

$$\int_{T_0}^{T_M} (s_1 - s_2)\, dT = \frac{2}{\sigma} \lim_{\psi \to \psi(\Gamma)} \frac{\partial F_2(\varphi(M), \psi)}{\partial \psi}.$$

We used the above-described method to measure weld temperature in the explosive welding of copper with constantan and manganin with constantan. The experimental scheme employed is depicted in Fig. 2. All of the plates were 2 mm thick. Figure 3 shows a typical oscillogram of the time dependence of voltage. It was obtained in the welding of copper with constantan in the following regime: collision angle $\gamma = 18°$; velocity of the contact point $v_C = 1000$ m/sec. The sweep rate was 20 μsec/cell, while sensitivity was 20 MV/cell.

Up to the moment the top plate came into contact with the bottom plate, the voltage recorded by the oscillograph was equal to zero. It increased sharply at the moment of contact. The slight dip of the front of V(t) before the plateau is due to the finiteness of the time of diffusion of the magnetic field from the weld to the free surface of the plate. As long as the measurement cable (point A) was connected far from the stagnation point O, all of the magnetic fields generated in the weld propagated to the free boundaries of the flow to the right of point A within the loop formed by the measurement leads. In this case, the recorded voltage was constant. As A approached O, some of the magnetic flux began to propagate to the left of A and voltage decreased. When A was located far to the right of O, the magnetic-field sources near A disappeared and the entire field propagated outside the measurement loop. The voltage ceased to depend on time (i.e., the position of point A relative to O) and entered a new plateau corresponding to the residual temperature of the weld.

Figure 4 shows the dependence of temperature in the weld on the distance to the stagnation point. This dependence was determined from analysis of the oscillograms. Table 1 shows maximum (top line) and residual (bottom line) temperatures in the weld for certain welding regimes in the cases of the welding of copper with constantan and manganin with constantan. The total temperature-measurement error due to the simplifying assumptions we made, the use of regularization methods, and the voltage-measurement error was estimated to be ±10%.

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LITERATURE CITED

1. I. D. Zakharenko, "Thermal regime in the weld zone during explosive welding," Fiz. Goreniya Vzryva, No. 2 (1971).
2. A. N. Mikhailov, A. N. Dremin, and V. P. Fettsov, "Temperature measurement in the weld zone during explosive welding," FGB, No. 4 (1976).
3. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Ill-Conditioned Problems [in Russian], Nauka, Moscow (1979).
4. V. V. Pai, "Determination of dynamic loading parameters in work-hardening and explosive welding," Physicomathematical Sciences Candidate Dissertation, Novosibirsk (1981).
5. V. V. Pai and V. A. Simonov, "Measurement of pressure in oblique collisions of metallic plates," Din. Sploshnoi Stredy, Collection of Scientific Works, Russian Academy of Sciences, Siberian Branch of the Institute of Hydrodynamics, No. 43 (1979).
6. H. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd edn., Oxford Univ. Press, NJ (1959).